QS015/2
Mathematics
Paper 2
Semester I
Session 2012/2013
2 hours

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Matematik
Kertas 2
Semester I
Sesi 2012/2013
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

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DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **17** halaman bercetak.
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SHAMMAL

1	Given that $f(x) = \begin{cases} 1+e^x, & x<1 \\ 1, & x=1 \\ 2-x, & x>1. \end{cases}$

Find $\lim\limits_{x\to1^-} f(x)$ and $\lim\limits_{x\to1^+} f(x)$. Does the $\lim\limits_{x\to1} f(x)$ exist? State your reason.

[5 *marks*]

2	Prove that $1+\tan 2\theta \tan\theta = \sec 2\theta$.

[6 *marks*]

3	Find the following limits:

(a)	$\lim\limits_{x\to\infty} \dfrac{2x^2+x-4}{1-x^2}$.

[3 *marks*]

(b)	$\lim\limits_{x\to2} \dfrac{3-\sqrt{x+7}}{x^2-4}$.

[4 *marks*]

4	Express $\dfrac{2x^3-7x^2+17x-19}{2x^2-7x+6}$ in the form of partial fractions.

[7 *marks*]

5 (a) Given that $f(x) = \begin{cases} \dfrac{\left|x^2 - x - 2\right|}{x^2 - 2x}, & x \neq 0, 2 \\ 0, & x = 2. \end{cases}$

Find the $\lim_{x \to 2} f(x)$. Is $f(x)$ continuous at $x = 2$?

[6 marks]

(b) A function $f(x)$ is defined by $f(x) = \begin{cases} \alpha x + 6, & x < 4 \\ x^2 + 2, & 4 \leq x < 6 \\ 2 - \beta x, & x \geq 6. \end{cases}$

Determine the values of the constants α and β if $f(x)$ is continuous.

[5 marks]

6 The polynomial $P(x) = 2x^3 + ax^2 + bx - 24$ has a factor $(x - 2)$ and a remainder 15 when divided by $(x + 3)$.

(a) Find the values of a and b.

[6 marks]

(b) Factorise $P(x)$ completely and find all zeroes of $P(x)$.

[6 marks]

7   Given $f(\theta) = 3\sin\theta - 2\cos\theta$.

    (a)   Express $f(\theta)$ in the form of $R\sin(\theta - \alpha)$, where $R > 0$, $0 \le \alpha \le \dfrac{\pi}{2}$.

        Hence, find the maximum and minimum values of $f(\theta)$.

                                                             [8 *marks*]

    (b)   Solve $f(\theta) = \sqrt{\dfrac{13}{2}}$ for $0^0 \le \theta \le 360^0$.

                                                             [4 *marks*]

8   (a)   Given that $y = \dfrac{1}{\sqrt{2x+1}}$.

        (i)   By using the first principle of derivative, find $\dfrac{dy}{dx}$.

                                                             [4 *marks*]

        (ii)   Find $\dfrac{d^2y}{dx^2}$.

                                                             [2 *marks*]

    (b)   Find $\dfrac{dy}{dx}$ of the following:

        (i) $y = e^{2x}\tan x$.

                                                             [2 *marks*]

        (ii) $y = x^{\sec x}$.

                                                            [4 *marks*]

9   (a)   A conical tank is of height 12 m and surface diameter 8 m. Water is pumped into the tank at the rate of 50 m^3/min. How fast is the water level increasing when the depth of the water is 6 m?

[6 *marks*]

(b)   A cylindrical container of radius r and height h has a constant volume V. The cost of the materials for the surface of both of its ends is twice the cost of its sides. State h in terms of r and V. Hence, find h and r in terms of V such that the cost is minimum.

[7 *marks*]

10 (a) Given $3y^2 - xy + x^2 = 3$. By using implicit differentiation,

(i) find the value of $\dfrac{dy}{dx}$ at $x = 1$.

[6 marks]

(ii) show that $\left(6y - x\right)\dfrac{d^2 y}{dx^2} + 6\left(\dfrac{dy}{dx}\right)^2 - 2\dfrac{dy}{dx} + 2 = 0$.

[2 marks]

(b) Consider the parametric equations

$$x = 3t - \frac{2}{t}, \quad y = 3t + \frac{2}{t} \quad \text{where } t \neq 0.$$

(i) Show that $\dfrac{dy}{dx} = 1 - \dfrac{4}{3t^2 + 2}$.

[3 marks]

(ii) Find $\dfrac{d^2 y}{dx^2}$ when $t = 1$.

[4 marks]

END OF QUESTION PAPER